

Accelerated expansion and the virial theorem

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When dark matter structures form and equilibrate they have to release a significant amount of energy in order to obey the virial theorem. Since dark matter is believed to be unable to radiate, this implies that some of the accreted dark matter particles must be ejected with high velocities. These ejected particles may then later hit other cosmological structures and deposit their momentum within these structures. This induces a pressure between the cosmological structures which opposes the effect of gravity and may therefore mimic a cosmological constant. We estimate the magnitude of this effect and find that it may be as large as the observed accelerated expansion. Our estimate is accurate only within a few orders of magnitude. It is therefore important to make a much more careful calculation of this redshift dependent effect, before beginning to interpret the observed accelerated expansion as a time dependent generalization of a cosmological constant.

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I. INTRODUCTION

The expansion of the universe has been observed to accelerate. This was first noted through the analysis of supernovae [1, 2] and more recently using cosmic microwave background and baryonic acoustic oscillations [3–5]. Most analyses have demonstrated that a cosmological constant is in good agreement with data [4, 6–8], however, one may easily imagine models which could allow for generalizations beyond a simple cosmological constant. Before we start parametrizing generalized models, or fitting new free parameters, it is important to carefully consider and remove known effects.

One effect, which has not received much attention in this context, arises from the merging between dark matter structures and is always present in a bottom-up formation scenario of cosmological structures. This effect induces an effective pressure between cosmological structures, which in principle may lead to systematic errors in the interpretation of the cause of the acceleration. We will now discuss the origin of this effect.

When a dark matter structure is formed and equilibrated, either through merging or accretion, it has to obey the virial theorem. The virial theorem quantifies the connection between the potential energy and the kinetic energy of the entire structure, $2K + W = 0$, where K is the total kinetic energy, and W is the total potential energy [9]. For instance, if a structure is formed by bringing cold particles in from infinity (i.e. initially $K_{\text{init}} = W_{\text{init}} = 0$), then the virial theorem implies that an energy corresponding to the resulting total kinetic energy, K , must be disposed of [10]. For non-radiative dark matter particles this means that a significant number of particles typically are ejected with high momentum.

In this Letter we explain and estimate this effect. We find that it indeed may have a significant contribution to the redshift dependent acceleration of the expansion. Since this is an unavoidable effect, we emphasize the need for a careful calculation of its affect on the cosmological expansion.

II. EFFECTIVE REPULSION

Let us consider a system of 3 equilibrated dark matter structures, 2 large and 1 small. The 2 large structures are placed at a large distance from each other, for instance at 100 times their virial radius. The small structure, called A, is placed close to one of the large structures, called B, for instance at 5 times the virial radius. A is given the circular velocity such that it is orbiting B in a stable circular orbit. The total force on the other large structure, called C, is given by the total gravity from A and B.

Now, let us consider a slightly different configuration, namely one where A is given the same speed as before, but directed towards B. This implies that after a few dynamical times A is essentially engulfed by B, and the 2 structures A and B will reach a new equilibrium configuration. The particles originating from A now sit at a deeper potential, and they therefore have higher kinetic energy on average.

At the end of this equilibration the virial theorem must hold, and therefore approximately half the total change in energy must have been disposed of. If the structures are composed of collisionless and non-radiative particles, then that energy can only be radiated away by “sacrificing” some of the incoming particles, or by increasing its size slightly. The ejected particles must leave the system with velocities above the escape velocity. These particles will generally leave the system in a wide cone along the axis of collision, however, for this discussion we just assume that they leave the system spherically.

What happens with these sacrificed particles with positive total energy? Some will just leave the system radially, and be of no further concern for now, however, some of them will happen to be directed towards the other large structure, C, and when they hit C they will transfer their momentum to it through standard gravitational effects like dynamical friction. The effect on C is therefore an effective pressure.

When we compare the 2 configurations above, then

we see that in both cases there will be a gravitational acceleration of C towards the combined system of A and B. However, in the second configuration there will be an additional pressure on C. If we knew nothing about the merger history of A and B, then we might interpret this extra acceleration of C as a negative gravitational effect. We will refer to this as the “*rejected acceleration*”.

If we were concerned with measuring the actual acceleration of the expansion of the universe, then it would be important to consider the magnitude of the effect described above, and if the effect would be non-negligible, then a careful subtraction must be done before we can ascribe the acceleration to e.g. a time varying cosmological constant.

III. HOW LARGE IS THE EFFECT

We now wish to make a rough estimate of the effect. Let us consider the universe today, and let us assume that half of the total mass has been assembled into structures, all with the mass of a galaxy, $M = 10^{12} M_{\odot}$. Naturally one should consider the full mass distribution, $N(M, z)$, however the argument below gives the same result for structures of $10^9 M_{\odot}$ and $10^{14} M_{\odot}$ within a factor of a few (largely from the difference in mass-concentration), so a proper integral over $N(M, z)$ is expected to give the same result within an order of magnitude.

Using a critical density of $\rho_c = 1.4 \times 10^{11} M_{\odot}/\text{Mpc}^3$, this means that the average distance between 2 structures is about 2.2 Mpc.

We wish to compare the *rejected acceleration* (that is, the effective acceleration induced by the sacrificed particles) to the acceleration which is generally believed to originate from a cosmological constant. The cosmological constant induces an acceleration of the order $GM_{\Lambda}(r)/r^2 \approx 9 \times 10^{-14} \text{ m/s}^2$ on a distance of $r = 2.2$ Mpc, where G is the gravitational constant, and $M_{\Lambda}(r)$ is $0.7 \rho_c$ integrated over a sphere of radius 2.2 Mpc. This simple estimate is, within a factor of 2, the same as one obtains when numerically solving the Friedmann equation. We therefore have that the acceleration from a cosmological constant is given by

$$a_{\Lambda}(2.2\text{Mpc}) \approx 9 \times 10^{-14} \text{ m/s}^2. \quad (1)$$

Next we consider the acceleration from the sacrificed particles. First it is important to remember that there is a time delay from the time when the particles were merged onto (and ejected from) one galaxy, and till they were absorbed by another galaxy. This retarded time depends on the typical distance between structures. If the ejected particles from a galaxy has of the order 200 km/s (similar to the peak dispersion in the galaxy) then a distance of 2.2 Mpc implies that the particles were ejected roughly 10^{10} yrs ago. We therefore see that only the particles ejected with at least 200 km/s (in addition to the kinetic energy to leave the potential) will reach the other galaxy today. The ejected energy may easily be

even higher or lower for some of the ejected particles. Numerical tests of the ejection mechanism seems to indicate that of the order 20–30% of the particles are ejected in a spherical cold collapse simulation [11], and between 10% and 40% in merger simulations [12].

Let us now discuss the rate at which the particles are ejected from the first galaxy. The merging rate has clearly not been a constant throughout the history of the universe, however, to get the order of magnitude we just simplify by a linear merging rate in time, such that $\delta M/\delta t = M/t_H = 10^{12} M_{\odot}/13.7 \text{ Gyr}$. We therefore use $\delta M/\delta t = 10^8 M_{\odot}/\text{Myr}$. To fulfill the virial theorem, half the changed energy will be ejected or used to increase the size of the system. The distribution is merger dependent, however, for this order of magnitude estimate, we simply consider a quarter of the energy to be ejected which implies that roughly $0.25 \delta M/\delta t$ will be ejected with $v = 200 \text{ km/s}$. The acceleration is therefore $dv/dt = v/M \times 0.25 \delta M/\delta t \approx 1.8 \times 10^{-13} \text{ m/s}^2$.

We finally have to consider how many of the ejected particles will deposit their momentum in another galaxy. If we consider a very long timescale, then all particles will eventually hit some galaxy. Equivalently, the one receiving galaxy may absorb particles from a range of previously merging structures. As a theoretical upper limit we therefore have, that all the ejected particles may be absorbed. However, many of these galaxies will lie at even larger distances, and the particles will therefore deposit their energy far into the future. In that case the velocities will be redshifted to lower values, and less energy will be deposited. Typically, the velocity must be redshifted below the escape velocity of the receiving structure [12].

We therefore conclude that the rejected acceleration must lie below the value found above

$$a_{\text{reject}}(2.2\text{Mpc}) < 1.8 \times 10^{-13} \text{ m/s}^2. \quad (2)$$

IV. DISCUSSIONS

We have seen above that the rejected acceleration is smaller than $1.8 \times 10^{-13} \text{ m/s}^2$, which happens to be of the same order of magnitude as the observed acceleration, $a_{\Lambda} \approx 9 \times 10^{-14} \text{ m/s}^2$ within a factor of a few.

We are here entertaining the view of the redshift being a true Doppler shift, since we are ascribing the acceleration to a changed velocity of the galaxies and other cosmological structure. This is possibly completely equivalent to the view of the expansion being a stretching of space [13]. This question is still actively debated (see a list of references in [14]) and we will not enter that discussion here. Similarly, it is also actively debated how the differences in an inhomogeneous space between a local accelerating space and a globally accelerating space will manifest themselves observationally (see e.g. [15, 16]).

We estimated above that the simplification from using one mass only gives a order of magnitude uncertainty. In

principle this can be calculated more accurately, by using the correct mass distribution, $N(M, z)$, e.g. from Press-Schechter theory. One complication that will arise is that particles ejected from high mass structures are difficult to absorb by smaller structures due to the higher particle velocities. In that sense there may be a larger effect of the rejected acceleration on larger structures.

The merging rate is both mass and redshift dependent, and therefore the merging rate (and hence the rejected acceleration today) may be somewhat different than the estimate used above.

Another point which was ignored above is that the mass of the structures were smaller at the time when the particles were ejected. For instance the $10^{12}M_{\odot}$ structure considered above would only have about have 1/4 of that mass 10^{10} yrs ago. We have also been using that a quarter of the energy must be ejected, however, if the first galaxy changes its size more than assumed during the merging, then it may hold more energy, and fewer particles need to be ejected, and vice versa.

A rather non-trivial point is the retarded time: the sacrificed particles will only be absorbed in other structures at a later time. This retardation will be different for different particles, depending on the mass of the structure they are first merged onto. The reason is that whereas small mass structures are formed earlier (hence giving a larger retarded time) then the sacrificed particles are ejected at lower velocities. This all implies that there should be a strong redshift dependence on the rejected acceleration. It is also interesting to note that the effect of rejected acceleration will diminish in the far future, when structure formation has been reduced for a long time, and all ejected particles have either been absorbed or their energies redshifted away.

To a first approximation this effect should be isotropic, as the observed acceleration is. However, there may be a slight anisotropic pressure near a large over-density where most of the merging happens along the filament. This possibility can be quantified by numerical simulations, by observing to which degree the ejected particles are emitted spherically or along the axes of merging.

One might ask if this effect should already have been observed by cosmological N-body simulations, and the answer is no. Most simulations are executed by separating the expansion of the universe from the local effects of structure formation. This means that the rejected acceleration cannot induce an extra acceleration of the expansion, but at best impose an extra radial pressure on all structures in present day simulations. This radial pressure is low, and will be very difficult to disentangle from the dynamics of the normally infalling particles. It is not known to me if a method to perform cosmological simulations exists, where one can dynamically include the effect of the cosmological constant: if it will be possible to include for instance a local density dependent acceleration due to a generalized cosmological constant, then it

will certainly also be possible to include the effect of the rejected acceleration, at least in principle.

Finally, a different point related to the retarded time is the velocity with which the particles are ejected from the original structures. There will naturally be a broad distribution of velocities, since not all particles are ejected at the same velocity. One therefore has to fold this velocity distribution of the ejected particles with the mass distribution, to find the full redshift dependence of this acceleration. One should also include the effect that different structures emit particles at different time, which may lead to an accumulation of the effect. For instance a $10^{10}M_{\odot}$ structure will reject particles earlier and slower, than a $2 \times 10^{10}M_{\odot}$ structure. Therefore, there may be an overlap of the arriving particles today, from the particles ejected at different times. Furthermore, above we merely used the proper distance today, however, in reality the relevant distance will be slightly shorter because of the expansion of the universe. Considering all these effects is a somewhat non-trivial calculation which we intend to address in the near future.

V. CONCLUSION

The observed accelerated expansion of the universe is generally believed to be caused by a cosmological constant, or some time dependent generalization hereof. In this brief note we point out a simply dynamical mechanism which may have a significant contribution to the acceleration.

The dynamical effect arises from merging of dark matter structures, where the virial theorem implies that a significant amount of kinetic energy must be carried away by ejected particles. These ejected particles will later deposit their momentum on other cosmological structures, inducing an effective accelerated expansion.

We estimate the effective accelerated expansion from the ejected particles to be of the same order of magnitude as the observed accelerated expansion of the universe. The simple estimate presented here is accurate only within a few orders of magnitude.

We emphasize that this dynamical effect is a well known physical effect which cannot be avoided. It is therefore something which needs to be calculated accurately and accounted for before measurements of a time varying cosmological constant will be trustworthy.

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